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HEAT TRANSFER ON A FLAT PLATE IMMERSED IN A LAMINAR FLOW OF A HEATED LIQUID

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The case of one-sided heat transfer to a flat plate immersed in a flow of heated liquid is examined. Equations are obtained for the liquid temperature distribution in the boundary layer, and for the heat flux from the liquid to the wall.

We shall examine heat transfer to a flat horizontal plate of finite thickness (Fig. 1), one surface of which is immersed in a laminar flow of a heated liquid. The opposite surface of the plate has constant temperature T_1 along its

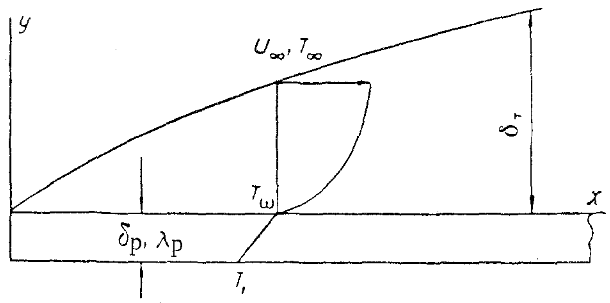

Fig. 1. Boundary layer on flat plate.

length, T_1 being less than the undisturbed temperature of the flow, T_∞. It is clear that the heat flux will depend both on the difference $T_\infty - T_1$, and on the thermal resistance of the plate. As the latter increases, the heat flux will tend to zero, and vice versa. At the same time, the temperature of the plate surface immersed in the liquid flow will tend to that of the free stream, and therefore the temperature distribution over the thermal boundary layer will also vary.

We must therefore evaluate the influence of the thermal resistance of the plate, both on the heat flux and on the liquid temperature distribution in the boundary layer.

If we assume that the physical properties of the liquid are independent of temperature, and also neglect the heat due to friction of the liquid on the plate, the equation for the temperature distribution in the boundary layer will have the form

$$\frac{d^2\vartheta}{d\eta^2} + \frac{\mathrm{Pr}}{2} f \frac{d\vartheta}{d\eta} = 0, \tag{1}$$

where

$$\vartheta = (T - T_1)/(T_\infty - T_1), \quad \eta = y\sqrt{U_\infty/\nu x},$$

and $f(\eta)$ is a function describing the velocity distribution in the boundary layer [1, 2].

The boundary conditions which the dimensionless temperature, ϑ, must satisfy are

$$\text{when} \quad \eta = 0 \quad \vartheta(0) - k_x \mathrm{Re}_x^{\frac{1}{2}} (d\vartheta/d\eta)_{\eta=0} = 0, \tag{2}$$

when $\eta = \infty$

$$\vartheta(\infty) = 1, \tag{3}$$

where

$$k_x = \lambda\delta_p/\lambda_p x; \quad \mathrm{Re}_x = U_\infty x/\nu.$$

Equation (2) expresses the condition of equality of the heat fluxes

$$\lambda\left(\frac{\partial T}{\partial y}\right)_{y=0} = \frac{\lambda_p}{\delta_p}(T_\omega - T_1). \tag{4}$$

Solving (1) with boundary conditions (2) and (3), we find the equation for the liquid temperature distribution in the boundary layer

$$(1 + 0.332\sqrt[3]{\mathrm{Pr}}\,k_x\,\mathrm{Re}_x^{1/2})\times$$
$$\times\,(T - T_\infty)/(T_1 - T_\infty) =$$
$$= \int\limits_{\xi=\eta}^{\infty}[f''(\xi)]^{\mathrm{Pr}}\,d\xi \Big/ \int\limits_{0}^{\infty}[f''(\xi)]^{\mathrm{Pr}}\,d\xi. \qquad (5)$$

In determining the constants of integration, we introduced the relations [1]

$$f''(0) = 0.332 \text{ and } (0.332)^{\mathrm{Pr}} \Big/ \int\limits_{0}^{\infty}[f''(\eta)]^{\mathrm{Pr}}d\eta = 0.332\sqrt[3]{\mathrm{Pr}}.$$

The relation on the right side of (5) is the temperature distribution found by Pohlhausen for the case of a cooled plate [1].

The heat flux through the plate is

$$q\,[(T_\infty - T_1)\,\lambda/x]^{-1} =$$
$$= 0.332\sqrt[3]{\mathrm{Pr}}\,\mathrm{Re}_x^{1/2}/(1 + 0.332\sqrt[3]{\mathrm{Pr}}\,k_x\,\mathrm{Re}_x^{1/2}). \qquad (6)$$

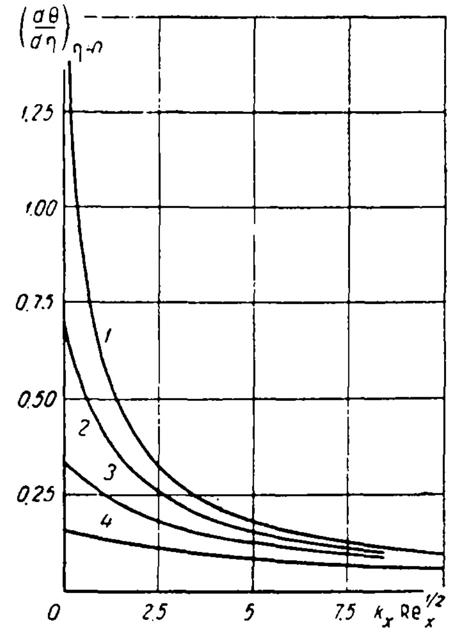

Fig. 2. Variation of temperature gradient at the plate surface as a function of the factor $k_x\mathrm{Re}_x^{1/2}$: 1—Pr = 100; 2—10; 3—1; 4—0.1.

It is clear from the results obtained that, if the thermal resistance of the plate increases, a flattening occurs in the temperature distribution profile over the boundary layer; in this case, $T_\omega \to T_\infty$. At the same time, as the Re number and the distance from the plate leading edge increase, the temperature profile also becomes flattened. This is because as x increases, the heat transfer process tends to equilibrium.

Furthermore, it is not hard to show that as $k_x \to 0$, $T_\omega \to T_1$. For this special case, Eq. (5) is identical to the equation found by Pohlhausen.

Figure 2 shows the variation of temperature gradient at the wall as a function of the factor $k_x\,\mathrm{Re}_x^{1/2}$ for various Pr numbers.

It is clear that, with increase of $k_x\mathrm{Re}_x^{1/2}$, the influence of the Pr number on the temperature gradient at the wall decreases. For constant thermal resistance and free stream velocity, increase of $k_x\mathrm{Re}_x^{1/2}$ corresponds to reduced x. It follows from this that at the beginning of the plate, where the boundary layer thickness is near zero, the influence of Pr on the temperature gradient may be neglected.

Our results are correct under conditions where the surface temperature T_ω varies weakly downstream. Moreover, it follows from (2) that the difference $T_\omega - T_\infty$ then retains its sign for all values of x.

Finally, it should be pointed out that the problem of formulating coupled systems, albeit in more general form, was first examined, as far as we know, in [3, 4].

NOTATION

λ—thermal conductivity of liquid; λ_p—thermal conductivity of plate; δ_p—plate thickness; ν—viscosity; U_∞—velocity of undisturbed flow.

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